Education Research and Perspectives, Vol.37, No.2

Using History to Enhance Student Learning and Attitudes in Singapore Mathematics Classrooms

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Numerous researchers have pointed to the potential benefits of providing students with a relevant historical context when introducing new mathematics concepts. This paper discusses the ways in which this approach may be applied to enhance student learning and attitudes in Singapore mathematics classrooms. While evidence on the efficacy of this approach remains limited, studies to date suggest that students who confront new mathematics concepts within a meaningful historical context will develop more positive attitudes toward the subject matter. Using history in mathematics teaching is likely also to help students recognise interrelationships between the disparate concepts to which they are introduced, and thus develop a more integrated view of the field as a whole. Relevant classroom activities, based on the Singapore junior college mathematics curriculum, are outlined.

Introduction

Comparative international data, such as those drawn from the *Trends in International Mathematics and Science Study* (TIMSS), indicate that Singaporean school and junior college students consistently perform well in mathematics (National Center for Education Statistics, 2009). Despite this, the proportions of junior college students in Singapore who opt to enrol in university mathematics courses are declining. In an attempt to account for this trend, several researchers have pointed to the negative attitudes that many students develop toward mathematics during their school years (e.g., Holton, Muller, Oikkonen, Valenzuela, & Ren, 2009). These claims add to those of previous researchers (e.g., Beaton et al.,1996; Ng, 2006), who have posed that it is not sufficient for Singapore educators to ensure that students acquire

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the skills they require to succeed in their post-schooling lives. Educators must also make effort to see that students develop deep understandings of, and positive attitudes toward, the subject matter as a whole, and will thus be motivated to engage in lifelong learning within the area.

One approach proposed to enhance students' attitudes toward mathematics is to introduce new concepts within a relevant historical context (Gulikers & Blom, 2001). Introducing new ideas in a meaningful framework can help students to see the value and utility of ideas, as well as helping them to see interrelationships between seemingly disparate concepts. Over the past three decades, researchers from various countries have discussed the possibility of using this approach to enhance student outcomes in mathematics classrooms. Fasanelli and Fauvel (2006) provide a comprehensive report on work done by numerous researchers in this area from the 1970s to the year 2000. Tzanaki (2008) also discusses some of this work.

In general, the studies reviewed previously have pointed to the benefits of using history in mathematics classrooms. Despite this, and despite the widespread international interest in the area (Fasanelli & Fauvel, 2006; Tzanaki, 2008), few educators in Singapore incorporate the use of history when introducing new mathematics concepts (Ahuja, Lim-Teo, & Lee, 1999; Ho, 2008). Although some textbooks make brief mention of significant mathematicians, mathematical history and events, these sections are typically presented as 'extra reading materials' which may be omitted both by teachers and by students. Courses or units on history of mathematics are also not offered to trainee teachers in Singapore (Kaur, Koay, & Yap, 2004), further underscoring the relatively low importance placed on this area.

This paper outlines 'typical' approaches taken to mathematics teaching in Singapore, based on previous empirical work. Some of the difficulties faced by students in learning mathematics are then discussed, along with ways in which these may be addressed by using history as context when introducing new concepts.

Empirical studies on the use of this approach are then reviewed, and classroom activities suitable for the junior college curriculum in Singapore are provided. The overarching goal of the paper was to highlight the ways in which providing historical context can be used to enhance students' learning and attitudes in mathematics at the school and junior college levels.

Mathematics Education in Singapore

Various studies have been conducted over the past 10 years to investigate how mathematics is taught in typical Singapore secondary schools (grades 7 to 10) and junior colleges (grades 11 and 12). For example, in one early study, Ahuja, Lim-Teo and Lee (1998) investigated Singaporean junior college teachers' views on their students' learning of calculus. In response to a questionnaire sent to all junior colleges in Singapore, most teachers (n = 91) commented that students' foundations in mathematics were weak due to the oversimplification of mathematical concepts that occurred in classroom teaching. Students were often given "loose pieces" of information that they did not know how to "piece together". Moreover, students tend to memorise formulae with minimal understanding of the concepts behind these formulae, and were not able to visualise the applications of these concepts and formulae in real life. Teachers also reported that students appeared neither motivated nor interested to learn calculus.

Numerous studies have also investigated directly how mathematics is taught within Singapore schools. For example, Yeo and Zhu (2005) reviewed the practices used across 18 primary five (Grade 5) and 19 secondary three (Grade 9) school classes in Singapore. Results indicated that lessons were mainly teacher-directed, with significant emphasis placed on routine procedural skills and basic concepts. Analysis of the classroom observations showed that students tended to learn via memorisation and regurgitation. In addition, Yeo and Zhu reported that students involved in their study were "passive" learners who seldom challenged what the teacher had taught. Teachers also seldom encouraged students to question their teaching.

Results from a series of other studies (e.g., Benedict & Kaur, 2005; Chang, Kaur, Koay, & Lee, 2001; Ho, 2005; Ho & Hedberg, 2005; Kaur, Low, & Benedict, 2007; Seah, Kaur, & Low, 2006) have corroborated these conclusions. Many of these studies have analysed video recordings of actual classroom lessons. A common observation made in these studies is that Singaporean mathematics teachers tend to use drill and practice as a core teaching strategy. In all the observed lessons, teachers gave students exercises and then worked through the solutions. The lessons observed were mostly teacher-centered, with most classroom time spent on practicing routine mathematics questions. As each of these studies involved only a few teacher participants, the results do not permit generalisations about how mathematics is taught at a national level. They do, however, give some indication of the teaching strategies that are favoured by many mathematics teachers in Singapore.

Attitudes Toward Mathematics in Singapore Schools

Several studies have also been conducted in Singapore on students' attitudes toward mathematics (see Lim, 2010). One of these studies (Fan et al., 2005) involved a survey of 1215 secondary students in eight schools. Fan et al. observed that while most students found mathematics important and were willing to learn mathematics, only 64% claimed that they would use mathematics outside school. In addition, students were not motivated to work on unfamiliar and challenging mathematics problems. Fan et al. suggested that students be given more opportunity to work on non-routine questions to improve their mathematics-related attitudes.

Lim-Teo, Ahuja and Lee (2000) conducted a study to assess junior college, polytechnic and university students' attitudes toward the learning of calculus. A total of seven junior colleges participated in this study. Among the 388 junior college students who were in their second year of study, 42.5% reported that they did not enjoy learning calculus. One reason given by these students for their lack of enjoyment was their poor foundation and understanding of the topic. In addition, 28.3% of junior college students found calculus lessons boring. Many students stated that they would have liked to have seen more real-life applications of mathematics, relevant either to their future work or to other subject areas. Some students also suggested that gaining an understanding of the history of how different concepts were developed within the field would increase their interest.

The above studies suggest that while Singapore students generally have positive attitudes toward the learning of mathematics, more can be done in classrooms to improve outcomes within certain domains. In particular, it appears that while students understand the importance of mathematics, and are willing to learn about the subject, they need more activities which put the tasks they do into a meaningful context. The latter activities should also emphasize to students the value and real-life relevance of the concepts to which they are introduced.

Using History as Context in Mathematics Classrooms

As reported in several studies (Calinger, 1996; Fauvel, 1991; Gulikers & Blom, 2001), the history of mathematics can potentially be used in classrooms to make learning more meaningful. History of mathematics refers to the processes of growth and development in mathematical knowledge and language (Otte, 2007). Burton (2003), Katz (1993a) and Eves (1990) define history of mathematics as a vast area of study which includes investigating sources of discoveries in mathematics, with an emphasis on (a) the problems mathematicians faced in the past, and (b) the knowledge developed by these mathematicians to solve their problems. It also includes investigating the achievements of significant mathematicians and their ideas.

Applications involving the use of history in mathematics classrooms have varied considerably. Some have emphasized the use of anecdotes and biographies of mathematicians in mathematics lessons (Bidwell, 1993; Higgins, 1944; Wilson & Chauvot, 2000). Others (e.g., Katz, 1993b) have argued that mathematics in education should involve the discussion of historical motivations for the development of each mathematical topic. Others still recommend the use of original sources to allow students to grasp the fundamentals of mathematical concepts more easily (Arcavi & Bruckheimer, 2000; Jahnke et al., 2000). A smaller group of authors has recommended teaching mathematics according to the chronological order in which the concepts were developed (e.g., Seltman & Seltman, 1978).

In essence, applications which have relied on providing history as context in mathematics classrooms have included one or more of four key elements: (1) the use of anecdotes and biographies of mathematicians; (2) the discussion of historical motivations for the development of content; (3) the use of original materials from historical sources; and/or (4) the teaching of a topic according to its chronological development in history. The next section discusses specific issues that students confront in mathematics which may be addressed by one or more of these practices. The section focuses first on how history may be used to address two major problems in mathematics learning: epistemological obstacles and misinterpretations of mathematical language. The section then discusses the ways in which history can make mathematics more interesting and relevant to students.

The idea of an *epistemological obstacle* (see Radford, Boero, & Vasco, 2000) was first introduced by Gaston Bachelard and subsequently used by Brousseau (1983, 1997) to refer to a piece of knowledge that is once used successfully to solve certain problems, but subsequently becomes an obstacle to learning when it fails to work in a different context. Consequently, the learning process is retarded by this piece of knowledge, which is already ingrained in the learner's mind. History of mathematics can be used to identify these epistemological obstacles in students and thus speed up the learning process (Moru, Persens, Breiteig, & Ndalichako, 2008). Moru et al. (2008) document various ways in which epistemological obstacles can be overcome via the use of history of mathematics. History of mathematics can also serve to highlight to teachers some of the mathematical misconceptions that students may have (Ernest, 1994b). This would enable teachers to employ more appropriate teaching strategies to facilitate students' learning.

Another major learning difficulty in mathematics arises from students' misinterpretations of mathematical language, including mathematical definitions and symbols (Selden & Selden, 2001). Tall and Vinner (1981) use the terms *concept definition* and *concept image* to distinguish a formal mathematical definition and a learner's own interpretation of a particular mathematical concept. Very often, a concept image will not be the same as a concept definition, even though the formal definition of the concept is clearly stated in the literature (Attorps, 2006).

Brown (1994) suggests that students' interpretations of mathematical language are based on their experience of usage, which is usually gained in classrooms. As concepts are often oversimplified in classroom teaching, students' classroom experiences may in themselves cause disparities between concept images and concept definitions (Freudenthal, 1991). This argument is supported by Furinghetti (2007), who posed that the actual meanings of mathematical concepts are often lost because mathematical knowledge is reconstructed in textbooks and by teachers. According to Tall (1991), mathematicians and teachers tend to break up a mathematical concept into smaller parts and sequence them logically from a mathematical expert's point of view. Students may not, however, be able to link the parts into a single concept. Hence, this "knowledge reconstruction" often leaves students confused about the mathematical concepts that they learn in schools (Siegel & Borasi, 1994).

To enable greater appreciation and understanding of mathematical language and concepts, it may be necessary to illustrate to students how mathematical language and concepts have been developed in history. For instance, the use of both dy/dx and $f'(x)$ to denote the first derivative of a function in many textbooks, is confusing to many students. The original mathematical publications written by Gottfried Wilhelm von Leibniz and Joseph Louis Lagrange make clear that dy/dx was used by Leibniz while $f'(x)$ was used by Lagrange, accounting for the prevalence of both notations in current works. Hence, the use of history offers a means to clarify students' misinterpretations of mathematical language.

Another consequence of the oversimplification of mathematical concepts in classrooms is that students often see mathematics as "a dead subject" (Lingard, 2001, p. 40), in which there is nothing left to be discovered. This problem is highlighted by Schwartz, Michal and Myles (1985) who lamented that:

> there is something odd about the way we teach mathematics in our schools. We make little or no provision for students to play an active and generative role in learning mathematics and we teach mathematics as if we expected that students will never have occasion to invent new mathematics" (p.1).

Several philosophers and mathematicians (Davis & Hersh, 1980; Ernest, 1997; Kitcher, 1983; Lakatos, 1976; Tymoczko, 1986) advocate a 'fallibilist' perspective in the teaching of mathematics. The fallibilist perspective asserts that school mathematics should not be taught as a rigid body of knowledge that is absolute and always correct, with no room for contribution by students. Students need to see how they can play a role in the development of mathematical knowledge, and the value of mathematics to their lives. Hence, for meaningful learning to take place in schools, students need to see the "human face of mathematics" (Ernest, 1994c, p. xi). To humanise mathematics is to relate mathematics to humankind and to identify the important role that mathematics has played in the development of human culture (Tymoczko, 1994).

Through history, students can appreciate the motivations behind the creations of knowledge by mathematicians, which have generally arisen from a need to solve a real-life problem in the past. Burton (1998) suggests that mathematics is learned by making connections to real-life phenomena. Although some mathematical concepts appear to be disconnected from real-life in modern times, history enables students to understand the need for the development of these concepts. For instance, by looking into the historical development of calculus, students realize that in the past, people were interested in solving physics problems on motion which led to the creation of differential calculus by Sir Isaac Newton (Grabiner, 1983). Kepler's desire to help wine traders estimate volumes of wine bottles similarly led to the formulation of integral calculus (Green, 1979).

By bringing into the classroom the anecdotes and biographies of mathematicians, as well as the different stages at which mathematical concepts were derived and theorems proved, students can see mathematics as a subject which has been “reinvented” by different people at various phases in history. Students can also see how mathematicians have erred and gone through difficulties before deriving their final concepts and theorems (Gulikers & Blom, 2001). D'Ambrosio (1995) argues that by investigating mathematicians’ cognitive activities in the past, students can better understand the role of human minds in constructing mathematical knowledge, and accept that they too can produce mathematical ideas that are original.

In addition, the use of history can highlight to students the role of mathematics in cultural anthropology. Kelley (2000) suggests that students can appreciate the artifacts of many cultures through history of mathematics. For instance, geometric properties such as symmetry and congruence can be found in the artworks of Africans and in Islamic mosaics. Kelley also uses history of mathematics to compare the mathematical traditions and inventions of the ancient Greek and Chinese. Ancient Chinese governments employed most of the country's mathematicians to work on activities such as building walls, moving and feeding vast armies, and farming, which resulted in the advancements in hydraulic engineering and navigation in Chinese mathematics seen today. In contrast, Ancient Greece had a wealthy ruling class that produced intellectuals, scholars and scientists who saw little need to apply their mathematical discoveries, and hence devoted their time to developing mathematics fundamentals.

Using history of mathematics as context can also provide opportunities for cross-curricular work between mathematics and other disciplines (Gulikers & Blom, 2001). For instance, as noted above, teaching the historical development of differential calculus to solve physics problems on motion allows students to see the connection between physics and mathematics (Grabiner, 1983). This is likely to arouse interest in students who find the learning of mathematics meaningless, yet appreciate its applications in other subjects as well as in real-life scenarios.

Example Classroom Activities

This section provides examples of the ways in which history can be incorporated within mathematics teaching. Burton (2003), Katz (1993a) and Eves (1990) agree that mathematics originated during the Stone Ages with the practical problems of counting and recording numbers. At that time, people needed to count their livestock and tally objects for barter trading. This counting system was, however, very primitive. As people's lives were harsh and difficult, they were more concerned with survival than with the development of mathematical knowledge. More sophisticated mathematical skills began to develop only when agricultural activities were initiated after 3000 BC. For the first time in history, some people such as kings, priests, merchants and scribes had leisure time to ponder over the mysteries of nature and science. In the process, they developed more sophisticated concepts in arithmetic and geometry. The present paper considers the development of mathematical knowledge after 3000BC as the starting point in the history of mathematics.

Smoryński (2008) offers a list of textbooks that chronicle the historical development of mathematics. Burton (2003) and Eves (1990) are two of the listed textbooks that are suitable for use in junior colleges. Besides textbooks, ideas on how history of mathematics can be used in classrooms abound in published papers. In this section, a review of research papers that are relevant to the GCE 'A' level mathematics curriculum is provided.

Gulikers and Blom (2001) provide an extensive list of studies in which history has been used within geometry lessons, and rate these studies based on (1) whether the amount of information provided in the article is sufficient for teachers to get acquainted with the subject and to feel comfortable teaching the subject; (2) the type of activities that can be done in class; (3) the extent of using original mathematical sources are used in the activities; and

(4) the amount of time teachers need to spend to prepare the activities suggested by the articles. Although the ratings provided in this review are based solely on the judgments of the authors, the paper provides a start-point for teachers who wish to use history in their geometry lessons.

The historical development of algebra is summarized by Katz and Barton (2007) who state that algebra originated from ancient geometrical problems such as the division of land. Original problems from historical sources are also used to explain how a number of algebraic formulae are derived. The contents of this paper are useful for explaining to students the motivations behind the derivation of specific algebraic formulae. Furthermore, the proofs of these formulae are presented in a way that is easy for junior college students to follow.

A detailed description of how a calculus course can be taught using a historical approach is presented by Katz (1993b). As the syllabus covered in Katz's course is well aligned with the GCE 'A' level curriculum, junior college teachers should find this article useful and relevant to their teaching.

Various other recommendations for the use of history in mathematics classes have appeared. For instance, Bidwell (1993) provides several interesting classroom activities that involve history of mathematics. Of particular relevance to the GCE 'A' level curriculum is an activity on complex numbers. Bidwell suggests using Geronimo Cardano's (1501 – 1576) problem[1] when introducing complex numbers to students. Ho (2008) describes

[1] Cardano was trying to find the values of x and y such that $x + y = 10$ and $xy = 40$. He obtained complex roots, which did not make sense to him at that time, but he proceeded to accept these complex roots as solutions to his problem. This gave rise to the notion of complex numbers which was further developed by other mathematicians such as Rafael Bombelli (1526 – 1573).

ways in which the historical development of the dot-product[2] can help students understand this abstract concept in vectors. In another example, Savizi (2007) describes the use of Al-Bruni's method to measure the earth's circumference as a practical application of trigonometry.

The above-mentioned classroom activities provide only a few suggestions on how history of mathematics can be used in junior college classes. A more extensive list of classroom activities that incorporate history of mathematics into lessons can be found in Fauvel and Van Maanen (2000).

Studies on Using History in Mathematics Classrooms

Numerous qualitative studies have investigated the impact of using history of mathematics on student outcomes. In one early example, Dittrich (1973) documented how mathematics could be taught to eleventh and twelfth grade students using biographies of mathematicians and historical sources. Dittrich reported that students showed greater interest in mathematics after they were exposed to these biographies. More recently, Ponza (1998) described the use of an interdisciplinary project on the life of Evariste Galois to motivate an ill-disciplined class of seventh grade students. Ponza reported a positive change in the attitudes of students toward mathematics learning.

Although these qualitative studies suggest that using history can improve mathematics outcomes in classrooms, they do not permit definitive conclusions to be drawn on the efficacy of this approach. Four controlled quantitative evaluations have, however, also been conducted to examine the impact of this approach on student outcomes. In an early study, McBride and Rollins (1977) examined the effects of using original derivations and proofs by famous mathematicians on the attitudes of college students taking

[2] The dot product of vectors is a concept taught in the GCE 'A' level mathematics curriculum and is defined as $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}|\cos\theta$, where **a** and **b** are vectors and θ is the angle between **a** and **b**.

an algebra course in the United States. A 2 x 2 factorial experiment with one factor for treatment and one factor for teacher was used. To minimize threats to internal validity, participants were randomly assigned to control and experimental groups, and a textbook (Swokowski, 1967) devoid of historical content was used in all classes. The results indicated a significant positive effect of the history intervention on students' attitudes toward mathematics.

Lit, Siu and Wong (2001) later conducted a quasi-experiment to investigate the effects of using history in teaching the Pythagorean Theorem on the attitudes of Secondary 2 (Grade 8) students in Hong Kong. Results indicated a significant positive effect of the history intervention on attitudes toward mathematics. There was, however, no significant positive effect on student achievement.

Ng (2006) examined the effects of an Ancient Chinese Mathematics Enrichment Program (ACMEP) on the academic achievement of 414 secondary two (Grade 8) students in Singapore. Classroom settings, student groupings and teaching approaches for both groups were kept similar between the two conditions. Results indicated that students exposed to the history material performed significantly better than those within a traditional control group.

Recognizing the dearth of research on the use of history of mathematics in Singapore, especially in the junior college and polytechnic level, Ho (2008) used an action research approach to investigate the effects of using history of algebra on the attitudes of 102 polytechnic students. Students' attitudes toward mathematics were measured in terms of beliefs, interests, confidence and perseverance. Ho reported higher scores in the areas of belief and perseverance in the post- than in the pre-test.

Despite the promising results presented by each of the studies described above, there is a need for further research in the area. For example, the study by McBride and Rollins (1977) was rigorous, but as the study was conducted with university-level students, its results may not generalize to school and junior college classrooms. The control and experimental groups in Lit et al.'s (2001) study differed significantly in their achievement levels at the pretest, which may have biased the study outcomes. In Ng's (2006) study, students were not assigned to control and experimental groups, but instead opted for one or the other, which again may have biased the study outcomes.

Thus, whilst the results reported by these researchers provide promising evidence to support the use of history as context in mathematics classrooms, the findings cannot be deemed to be conclusive at this point. Further research is needed to pinpoint the precise impact of this approach on student learning outcomes and attitudes. The findings do, nonetheless suggest that the approach may have positive effects in both the cognitive and affective domains, at least under some circumstances.

Conclusion

The use of history as context in mathematics classrooms has been recommended by various educators over the past few decades (Fasanelli & Fauvel, 2006). Despite this, the approach is rarely used in Singapore classrooms. Some researchers have urged caution in the application of the methods described in this paper. For example, Grattan-Guinness (1978) argued that it is undesirable for students to be "bogged down" by complex historical issues, and suggests that history of mathematics needs to be simplified before teachers use it in their teaching.

Fowler (1991) and Fauvel (1991) also suggested that there is a need to equip teachers with adequate knowledge and skills to identify appropriate historical sources to include into their teaching. These concerns were echoed by Furinghetti (2000) and Schubring et al. (2000), who posed that it may be necessary for teachers to receive additional training through courses on history of mathematics to implement the approach.

Although evidence remains scarce, preliminary empirical results suggest that using history as context in mathematics classrooms may enhance student outcomes, both in the affective and in the cognitive domains. These results suggest that, if the above conditions can be met, the use of history this approach may provide a viable solution to some of the problems raised in recent times with respect to mathematics teaching in Singapore schools.

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